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THE TEMPERATURE INNER-LAW AND HEAT TRANSFER FOR TURBULENT AIR FLOW IN A VERTICAL SQUARE DUCT

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Abstract—The temperature near heated walls of a vertical aluminum square duct has been measured for turbulent air flow. A miniature thermocouple probe was developed for the investigation, as well as special techniques for interpreting the results close to a wall. Moderate heat-transfer rates were employed and it was found that the temperature inner-law parameters correlated the profiles at all positions around the duct wall if the local values of the wall shear stress and wall heat flux were specified in the correlation.

The local wall heat-transfer rates were obtained by a recently described heat-flux probe. For the wall shear stress determination Preston tube data were employed. The distributions of wall shear stress and wall heat flux were specified by a single curve over the range of Reynolds and Nusselt numbers employed in the investigations.

NOMENCLATURE

c_f, shear stress coefficient, $c_f = \dfrac{2\tau_0}{\rho U_{av}^2}$ [dimensionless];
c_p, specific heat at constant pressure [Btu/lb-degF];
g, gravitational acceleration [ft/s^2];
l_e, correction factor for thermal conduction in a thermocouple probe [ft];
Nu, Nusselt number [dimensionless];
Pr, Prandtl number [dimensionless];
Re, Reynolds number [dimensionless];
q, heat-flux [Btu/h-ft^2];
t, temperature [°F];
t^+, inner-law temperature [dimensionless];
u, velocity [ft/s];
u^+, inner-law velocity [dimensionless];
u_*, wall shear velocity, $u_* = \sqrt{(\tau_0/\rho_0)}$ [ft/s];
y, distance from wall [ft];
y^+, inner-law dimensionless distance from wall [dimensionless].

Greek symbols
ν, kinematic viscosity [ft^2/s];
ρ, fluid density [lb-s^2/ft^4];
τ, shear stress, $\tau = \rho\nu \dfrac{du}{dy}$ [lb/ft^2];
φ, velocity inner-law function [dimensionless];
φ', temperature inner-law function [dimensionless];
φ'_r, heat-flux probe calibration [dimensionless].

Subscripts
av, average value;
m, measured value of velocity or temperature;
loc, local wall value;
o, wall value.

1. INTRODUCTION

EXTENSIVE evaluations exist for turbulent air flow near flat or cylindrical walls, both for momentum and heat transfer, and in particular for rigid impermeable surfaces. These geometries are characterized by a common simplicity in the structure of the flow since lateral mean velocity

4C

and lateral turbulent velocity correlations do not exist. Hence these flows are classified as two-dimensional.

For moderate pressure gradients in the direction of motion two-dimensional flows are successfully described in the wall region by wall parameters obtained by mixing length theories. The wall shear stress and perpendicular distance from the wall suffice to describe the velocity profile, while in addition the description of the temperature profile requires the wall heat-transfer rate [1, 2].

An obvious extension of the wall laws for momentum and heat transfer is to enquire about the mechanisms of flow near walls of more complicated geometry, and in particular about the pertinent correlation parameters. This paper will consider the case of walls that provide non-uniform flow conditions in a lateral direction, but which retain uniform properties in the principal direction of flow. A wide class of non-circular conduits is admissible, including the presently investigated square duct.

2. ANALYSES OF TURBULENT AIR FLOW NEAR WALLS

2.1 *Momentum transfer*

As mentioned in the introduction, extensive evaluations exist for turbulent air flow near flat or cylindrical walls. The velocity profile is expressed as a dimensionless ratio of the wall shear velocity and in terms of the dimensionless distance from the wall [1, 2],

$$\frac{u_y}{u_*} = u^+ = \varphi\frac{(y\,u_*)}{\nu} = \varphi(y^+). \qquad (1)$$

In the viscous sublayer, $0 \leqslant y^+ \leqslant 5$, inertial forces are negligible, the velocity gradient is linear, and

$$u^+ = y^+. \qquad (2)$$

A buffer layer separates the viscous sublayer from the fully turbulent flow, and is characterized by the presence of viscous and inertial forces in the momentum balance. The momentum balance is still satisfied by a shear stress equal to the wall value, as is the case for a small region of the fully turbulent flow just beyond the buffer layer. Reynolds stresses which control the momentum transfer in this fully turbulent layer can be predicted by mixing length theories which in turn provide a logarithmic velocity law for this so called inner-law layer, namely

$$\begin{aligned} u^+ &= A \ln y^+ + B, \\ &= 5{\cdot}67 \log_{10} y^+ + 5{\cdot}51, \end{aligned} \qquad (3)$$

after Leutheusser [5].

Although the velocity inner law was developed for two dimensional flows Leutheusser demonstrated its validity in a square duct for all points around the perimeter that could be measured by probes. That is, a very small region in the corners could not be tested. The significant innovation was that local values of the wall shear velocity should be used in equation (3) for each position around the perimeter.

An inspection of the mechanisms and assumptions governing the velocity inner-law reveals that convective terms must be absent from the momentum balance. This condition which is satisfied by a thin layer in two-dimensional flows is also satisfied by a similar layer in a square duct even though the flow pattern contains secondary currents, which convect significant quantities of momentum toward the corners of the duct [6, 7]. However, these currents cannot be significant in the near wall region where all mean velocity components are small, so that the inner-law requirements are satisfied. Thus the parameters governing the velocity inner-law for two dimensional flows are valid for square ducts and other non-circular conduits.

2.2 *Heat transfer*

The temperature profile of turbulent air flow near flat or cylindrical walls is expressed by a temperature inner-law which, for a moderate temperature difference becomes, for

$$\frac{q}{\tau} = \frac{q_w}{\tau_w} = \text{const.} \tag{4a}$$

$$Pr = 1{\cdot}0 \tag{4b}$$

$$\text{Either: (1) } Pr = 1{\cdot}0 \text{ or} \tag{4c}$$

$$\text{(2) } \begin{matrix} \varepsilon_m \gg \nu \\ \varepsilon_q \gg \alpha \end{matrix} \text{ in the turbulent core.}$$

Deissler [2, 3] as well as von Kármán [4] used these assumptions in the analytical development of universal profiles which can be expressed as

$$\rho_0 g \, c_{p0} \, u_* \frac{t_0 - t_y}{q} = t^+ = \varphi'(y^+, Pr). \tag{4}$$

In the viscous sublayer heat is transferred by molecular conduction, the temperature gradient is constant, and

$$t^+ = y^+ \, Pr. \tag{5}$$

In the inner law region mixing length theories indicate that the temperature profile is described as

$$t^+ = C \ln y^+ + D. \tag{6}$$

If the Prandtl number of the fluid is one and if the transfer of momentum and heat by the turbulent eddies is equal, then the velocity and temperature profile must coincide. Thus C and D are equal to A and B respectively. If the fluid has a Prandtl number other than one but the turbulence structure transports heat and momentum identically, then C has the value of A, but D is dependent upon the Prandtl number, since the value of the temperature profile is influenced in the viscous and buffer layers. For Prandtl numbers less than one D must be less than B, etc. This trend can be seen in equation five. In particular the temperature inner-law obtained by Johnk and Honratty [8] for moderate heat transfer to air flow in a pipe is

$$t^+ = 5{\cdot}1 \log_{10} y^+ + 3{\cdot}3. \tag{7}$$

This value of C is slightly different than A, but can be replaced by A if D is modified.

It is evident that the temperature inner-law depends upon a balance of the thermal equation which excludes the convection of heat by the mean flow, just as convective effects were excluded from the momentum balance and velocity inner-law. Following the discussion of the velocity inner-law it is also reasonable to assume that the temperature inner-law will describe the temperature profiles in a square duct if local values of the wall shear stress and wall heat-transfer rate are used in the correlation. This modification will be presently evaluated for a square duct, with the knowledge that if valid it will apply to other noncircular conduits.

3. DESCRIPTION OF EXPERIMENTAL EQUIPMENT AND TECHNIQUES

3.1 *Heated square aluminum duct*

A square duct was fabricated from architectural grade extruded aluminum, with inside dimensions of 3·767 × 3·767 in, with sharp inside corners, and with a wall thickness of 0·188 in. Two 15-ft lengths were bolted together to give a test section length of 30 ft, or 96 hydraulic diameter. The duct was mounted vertically to eliminate velocity and temperature distortions at the moderate Reynolds numbers achieved in the test programme. Thermal expansion was accommodated by suspending the assembly from the top and by providing an extensible rubber sleeve between the fan and settling chamber, see Fig. 1.

The duct was heated by 10 circuits of 0·5 × 0·005-in thick by 50-ft long Chromel "C" heater tape. For the heat rates employed the heater tapes were connected together as pairs in series to external rheostats which were connected to a d.c. generator driven by a 3-phase 550 V 15 hp motor. The motor–generator set was very stable with no variations in the supply voltage being detected. The aluminum duct was insulated by a layer of fiberglass cloth and high temperature varnish before the heater tapes were spirally wrapped onto the duct.

A guard heater of 0·125 × 0·005-in thick Chromel "A" heater tape was wrapped on the final 4 in of the duct to compensate for end losses.

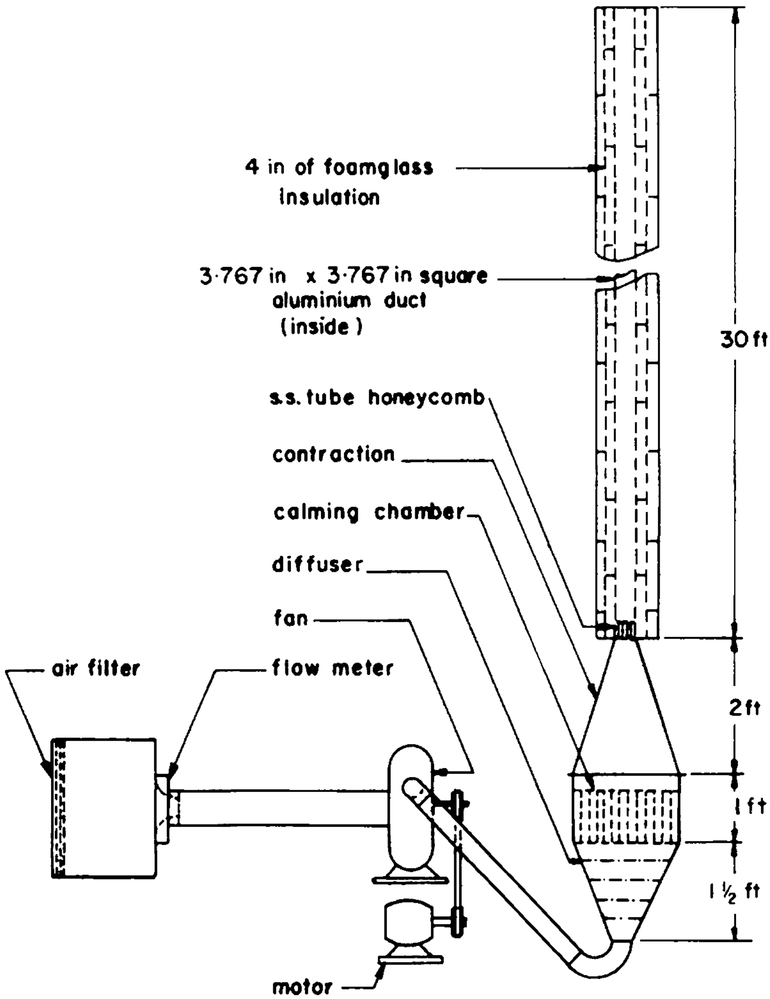

FIG. 1. Details of the heated square duct.

The guard heater was independently controlled by a rheostat, with power being provided by the d.c. generator.

The heater tapes were covered by a layer of fiberglass cloth and by two layers of 2-in thick foamglass insulation.

The heater circuits were adjusted to give a constant temperature drop from the walls of the duct to the centerline over the 20 to 30 hydraulic diameters of established flow before the exit. Thus the longitudinal temperature gradients along the walls and along the centerline were identical as determined by wall and centerline thermocouples positioned along the final 15 hydraulic diameters.

Average heat-transfer rates were determined from the rate of temperature increase of the air in the longitudinal direction and from the discharge of air. An independent heat balance based upon the rate of electrical dissipation and calculated heat loss gave very good agreement. Discharge was determined by a calibrated flow meter, with variations in the rate being controlled by a damper positioned at the fan outlet.

3.2 *Shear stress measurements*

The local wall shear stresses were determined by Leutheusser who used calibrated Preston tubes, and a Chattock manometer [5].

3.3 *Temperature evaluation*

Temperature readings were recorded by a Photovolt temperature recorder. The recorder had a full scale deflection of 0·5 mV for 10 in of chart width, and a full scale response of 0·75 s. All temperature measurements were made with copper–constantan thermocouples. Highest accuracy was obtained by recording the temperature difference between the walls of the duct, and a temperature probe. The rates of heat transfer through the $\frac{3}{16}$-in aluminium walls were low enough, with respect to the thermal conductivity of aluminum, to ensure that a uniform wall temperature existed around the duct perimeter, at each axial station. Hence for a given axial position in the square duct, all temperature and heat flux measurements could be referred to the common wall temperature.

3.4 *Wall heat flux measurements*

The local wall heat-transfer rates were determined with a heat flux probe. The probe consisted of a thermocouple junction placed at the end of a square-ended tube. The probe geometry was similar to that of a Preston tube. The probe was positioned parallel to the walls of the duct and was directed in the upstream direction for all tests. Best accuracy was achieved by locating the probe 0·005 in away from the wall thereby eliminating conduction errors to the probe from the duct walls. The temperature difference obtained between the probe and wall thermocouples was calibrated and found to be a function of the temperature inner-law parameters such that

$$t_m^+ = \frac{t_0 - t_m}{\dfrac{q_{\text{loc}}}{\rho_0 g_0 c_{p0} u_{*\text{loc}}}} = \varphi'_t \left(\frac{u_{*\text{loc}}\, d}{\nu}\right). \tag{8}$$

Then the calibrated probe was used to provide local values of the heat flux by first obtaining the local shear velocity with a Preston tube, and also by noting the temperature difference between the probe and wall thermocouples. Further details of the heat flux probe can be obtained in [9].

3.5 *Temperature measurements in the inner-law region*

A miniature copper–constantan thermocouple junction was formed and positioned in a streamlined probe for temperature measurements in the inner-law region, Fig. 2. Great care was taken at each stage of the manufacture to ensure that thermal conduction errors and aerodynamic disturbances would be minimized.

The thermocouple junction was formed from 0·0031 in wires by a submerged arc welding process. The thermocouple wires were twisted together and then submerged in light mineral oil until contact was made with a layer of mercury. A d.c. electrical circuit was thereby closed, such that the resulting arc produced a strong maleable junction. The junction was assembled in stainless steel tubing with a suitable epoxy resin filler. Care was taken to fashion an aerodynamic probe, and to isolate the thermocouple junction from the tube by more than 100 thermocouple wire diameters.

A typical temperature profile obtained by the probe is shown in Fig. 3. It is evident that the uncorrected readings were influenced still by conduction errors, particularly in the viscous sublayer and buffer layers. Corrections for thermal conduction were based upon the temperature gradient in the fluid at the thermocouple position, expressed as

$$\Delta t_e = l_e \left(\frac{\mathrm{d}t}{\mathrm{d}y}\right)_y, \tag{9}$$

where

$$(t_0 - t_y)_{\text{corrected}} = (t_0 - t_y)_{\text{measured}} \pm (\Delta t_e).$$

The sign of Δt_e was determined by the direction of heat transfer, and was negative for heated walls. The magnitude of l_e the thermocouple

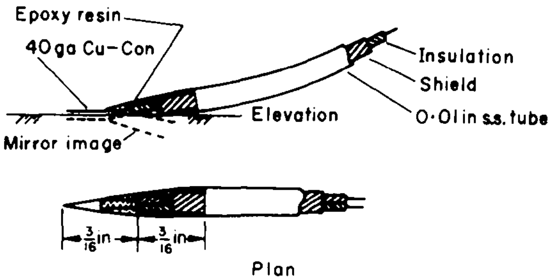

FIG. 2. Details of the inner-law temperature probe.

conduction parameter was determined by noting the discrepancy between the calculated and measured temperatures in the viscous sublayer, since a constant temperature gradient was assumed. The correction Δt_e was obtained at all positions outside of the viscous sublayer by successive approximations. First the correction was estimated with the measured temperature gradient. Subsequent corrections were made upon the corrected profile, until Δt_e became constant. For the described thermocouple Δt_e was negligible for $y^+ > 40$. For $10 < y^+ < 40$ one approximation for Δt_e was satisfactory and for $y^+ < 10$ two approximations were sufficient.

4. EXPERIMENTAL RESULTS

4.1 *Local wall heat transfer and shear stress measurements*

Wall shear stress and heat-transfer data are presented in Fig. 4. The wall shear stress data are by Leutheusser, from a wooden duct, using calibrated Preston tubes [5]. The heat transfer data are obtained with the heat flux probe in the heated aluminum duct. Both sets of results are expressed as dimensionless ratios of the local to average wall values, for a traverse from the mid point of a wall to one of the adjacent corners. Due to symmetry the wall values obtained in this traverse are repeated eight times about the duct perimeter and thus completely define the respective distributions. No variations occurred in the direction of flow since adequate development lengths were available, 288 equivalent duct diameters for the shear stress data, and 96 equivalent duct diameters for the heat-transfer data. The distribution of wall shear stress and wall heat flux are specified by a single curve over the range of Reynolds and Nusselt numbers to an accuracy of ± 2 per cent.

An interpretation of the results follows if the

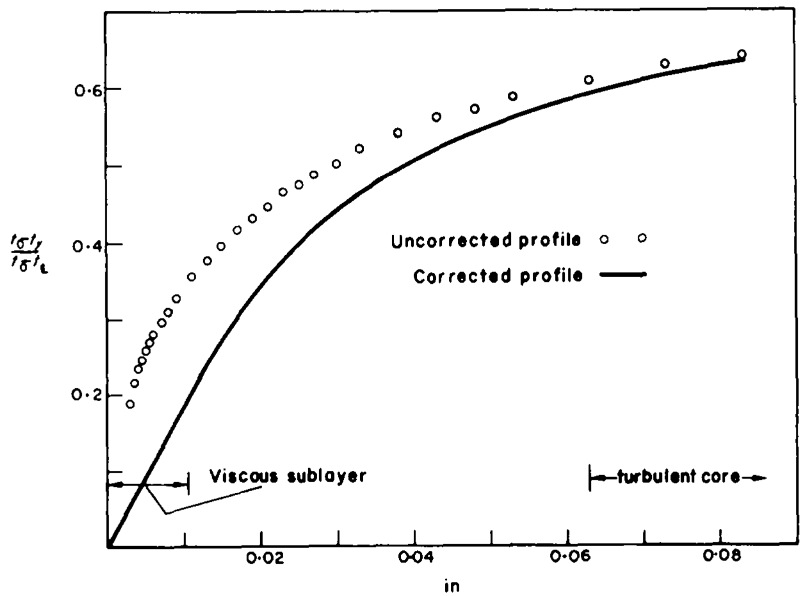

FIG. 3. Corrected and uncorrected temperature profiles.

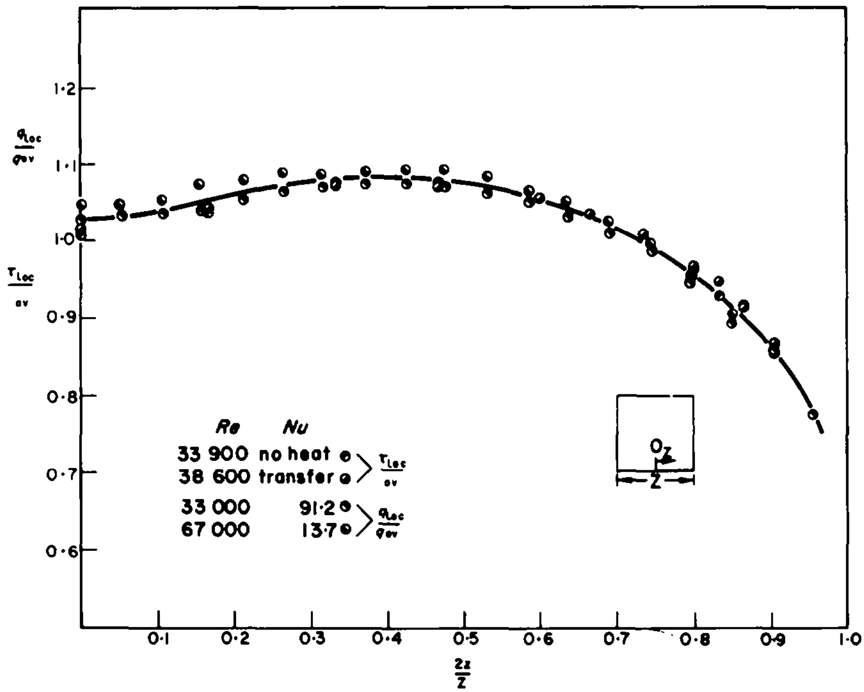

FIG. 4. Comparison of measurements of local wall shear stress and local heat transfer.

basic flow patterns in a square duct are considered [6, 7]. Data from [6] for the lines of constant axial velocity, isovels, are presented in Fig. 5, while data for the secondary current patterns are given in Fig. 6. In [6] and [7] it is shown that secondary currents act as an effective mechanism to transfer momentum from the center of a square duct to the corner regions. So much so in fact, that the resulting velocity gradients near the wall and hence the wall shear stress are largest at positions partway between the corners and the mid point of the wall. This effect is evident from the distribution of the isovels as well.

A logical interpretation of the wall heat-transfer distribution follows if it is assumed that the mixing properties of the secondary currents have a similar effect on heat transfer as for momentum transfer. As a result, since fluid from the center of a square duct is brought into closest proximity to a wall at the above mentioned positions, therefore it must produce the largest wall temperature gradients and hence the largest wall heat-transfer rates at these positions. Hence the distributions for wall shear stress and wall heat transfer must be similar.

A further conclusion is that the mixing action of the secondary currents can be likened to the mixing action of two dimensional turbulent flows such as occur in pipes and on flat plates. For such flows the concept of eddy diffusivities of the turbulence for the transfer of heat and momentum are convenient with the ratio of the eddy diffusivities defining a turbulent Prandtl number. For a large class of flows this value has been found to be near unity. Consider now the more complicated flow in a square duct, with mixing by secondary currents. It is logical to inquire if an effective turbulent Prandtl number can be inferred from the ratio of heat and momentum transfer arising from the diffusion of the turbulent eddies, and the mixing of the secondary currents. If length scales are prescribed in an attempt to define mixing lengths,

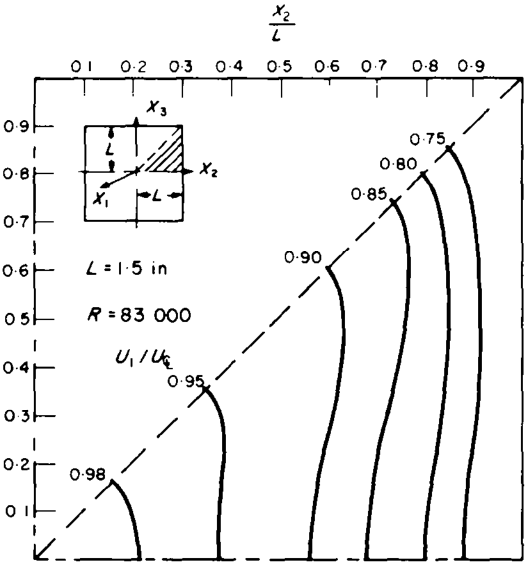

FIG. 5. Isovels for a square duct.

we see that the same scales must apply to both processes. But the best recourse is to observe the ratio of average friction and heat-transfer coefficients for a square duct to those of a pipe. The data of Fig. 7 indicate that the ratio of average friction and heat-transfer coefficients for a square duct are approximately the same as for a pipe. This is true even though both values for the square duct are slightly lower, due to the

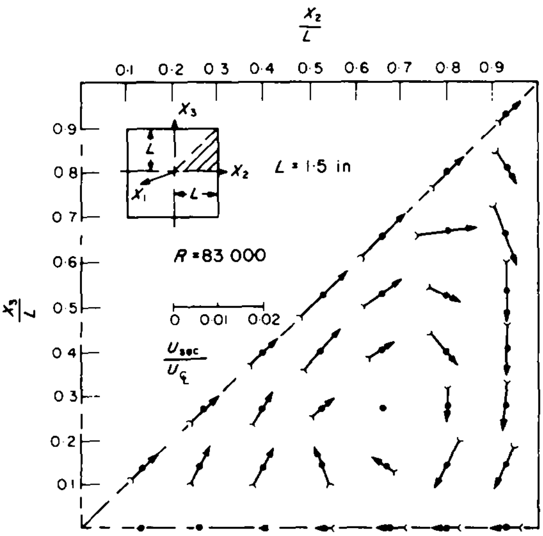

FIG. 6. Secondary currents for a square duct

regions of stagnation in the duct corners. Thus it is apparent that the secondary currents although a complicating process in square duct flow result in an effective turbulent Prandtl number for heat transfer which is identical to that for pipe flow and is near unity. Identical conclusions would appear to be valid for other flows containing secondary currents.

4.2 *Inner-law temperature measurements*

The temperature profiles for the square duct are plotted for various Nusselt and Reynolds numbers, and for several positions around the perimeter in Fig. 8. Inner-law parameters are employed, with the significant innovation that local values of the wall shear stress and wall heat-transfer rate are used in the correlation. A single curve describes all of the data when local wall parameters are used to correlate the temperature profiles around the duct perimeter. A similar conclusion was reached by Leutheusser regarding the inner-law velocity profiles and

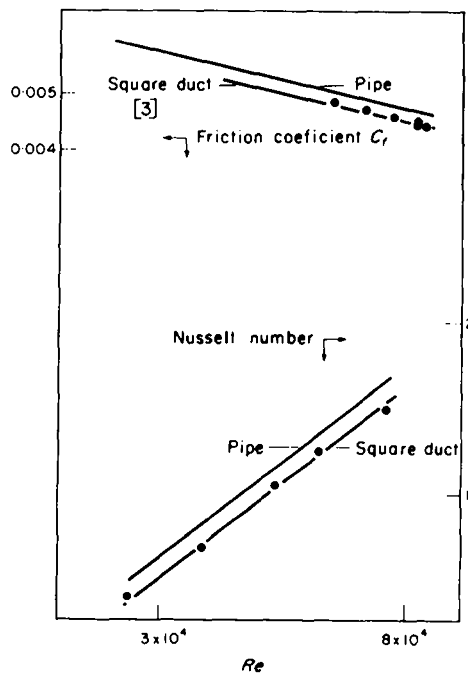

FIG. 7. Comparison of shear coefficients and Nusselt numbers for pipes and square ducts.

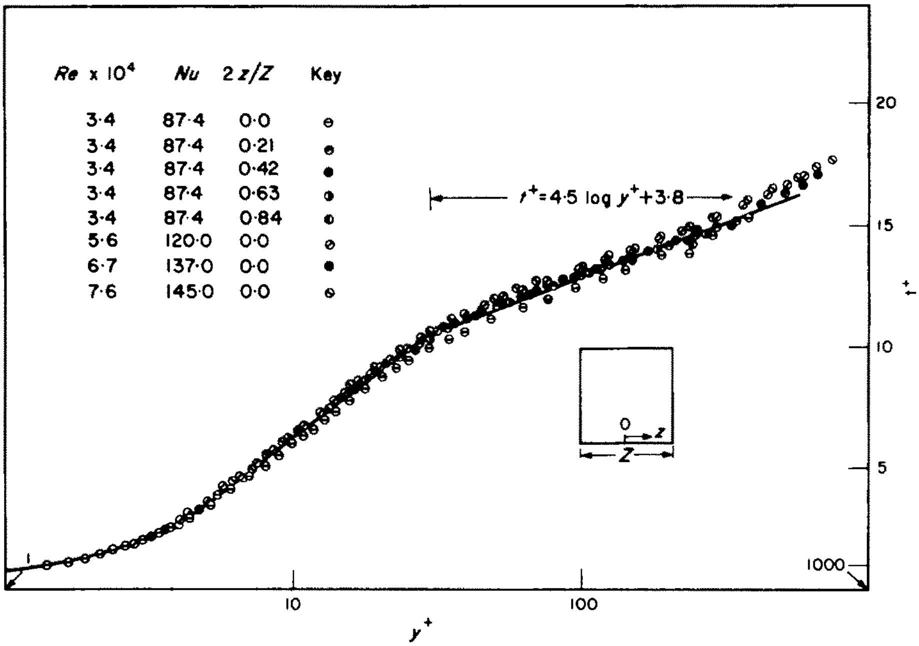

FIG. 8. Inner-law temperature profiles for a square duct.

local wall shear stress. For the square duct the logarithmic portion of the temperature inner-law was described by

$$t^+ = 4{\cdot}5 \log_{10} y^+ + 3{\cdot}8, \qquad (10)$$

to an accuracy of ± 4 per cent.

5. CONCLUSIONS

The temperature near heated walls of a vertical aluminum square duct has been measured for turbulent air flow. For the moderate heat-transfer rates employed it was found that the temperature profiles could be described by the temperature inner law parameters developed for two dimensional flows, if local values of the wall shear stress and wall heat-transfer rate were used in the correlation.

The local wall heat-transfer rates were obtained by a recently described heat flux probe, and compared to available local wall shear stress data. The distributions of the wall shear stress and wall heat transfer were specified by a single curve over the range of Reynolds and Nusselt numbers, to an accuracy of ± 2 per cent.

A further consequence of the evaluation was that the more complicated flow arising in non-circular conduits appeared to possess an equally effective mechanism for the transfer of heat as for momentum, in the fully turbulent core.

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Résumé— La température au voisinage des parois chauffées d'un conduit vertical en aluminium de section carrée a été mesurée pour un écoulement turbulent d'air. Une sonde miniature à thermocouple a été imaginée pour cette étude, en même temps que des techniques spéciales pour l'interprétation des résultats près d'une paroi. On utilise des flux de transport de chaleur modérés et l'on a trouvé que les paramètres de la loi intérieure de température corrélaient les profils à toutes les positions tout autour de la paroi du conduit, si les valeurs locales de la contraintes tangentielle pariétale et du flux de chaleur à la paroi étaient spécifiées dans la corrélation.

Les densités de flux de chaleur pariétales ont été obtenues à l'aide d'une sonde de flux de chaleur décrite récemment. On a utilisé les résultats d'un tube de Preston pour la détermination de la contrainte tangentielle pariétale. Les distributions de la contrainte tangentielle pariétale et du flux de chaleur à la paroi ont été représentées à l'aide d'une courbe unique dans les gammes des nombres de Reynolds et de Nusselt utilisés dans les recherches.

Zusammenfassung—Die Temperatur nahe beheizter Wände in einem senkrechten Aluminiumkanal von Rechteckquerschnitt wurde für turbulente Luftströmung gemessen. Neben speziellen Auswertmethoden für wandnahe Messungen wurde ein Miniaturthermoelement für die Untersuchung entwickelt. Geringe Wärmetransportleistungen wurden aufgebracht und dabei zeigte sich, dass Parameter für die Temperaturverteilung die Profile für alle Wandstellen gut wiedergaben, wenn die örtlichen Werte der Wandschubspannung und des Wärmeflusses in der Korrelation vorgegeben wurden.

Die örtlichen Wärmeübergangsleistungen wurden mit einem kürzlich beschriebenen Wärmeflussmesser erhalten. Zur Bestimmung der Wandschubspannung dienten Werte des Prestonrohres. Die Verteilung von Wandschubspannung und Wärmefluss wurde in einer einzigen Kurve wiedergegeben für den Bereich von Reynolds- und Nusseltzahlen der den Untersuchungen zugrundelag.

Аннотация—В турбулентном потоке воздуха измерялась температура вблизи нагретых стенок вертикального алюминиевого канала квадратного сечения. Для исследований была сконструирована микротермопара и разработаны способы расшифровки результатов вблизи стенки. Использовались средние скорости теплообмена, и было найдено, что параметры температурного поля внутри тела определяют профили во всех положениях на стенке канала, если установлена связь между локальными значениями на стенке напряжения трения и теплового потока.

Локальные зависимости теплообмена на стенке получены с помощью недавно описанного тепломера. Для определения напряжения трения на стенке для трубы использовались данные Престона. Распределения напряжения трения и теплового потока на стенке определялись единственной кривой в исследуемом диапазоне чисел Рейнольдса и Нуссельта.